The log-normal distribution from Non-Gibrat's law in the middle scale region of profits

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Abstract

Employing profits data of Japanese firms in 2003–2005, we kinematically exhibit the static log-normal distribution in the middle scale region. In the derivation, a Non-Gibrat's law under the detailed balance is adopted together with following two approximations. Firstly, the probability density function of profits growth rate is described as a tent-shaped exponential function. Secondly, the value of the origin of the growth rate distribution divided into bins is constant. The derivation is confirmed in the database consistently.

This static procedure is applied to a quasi-static system. We dynamically describe a quasi-static log-normal distribution in the middle scale region. In the derivation, a Non-Gibrat's law under the detailed quasi-balance is adopted together with two approximations confirmed in the static system. The resultant distribution is power-law with varying Pareto index in the large scale region and the quasi-static log-normal distribution in the middle scale region. In the distribution, not only the change of Pareto index but also the change of the variance of the log-normal distribution depends on the parameter of the detailed quasi-balance. As a result, Pareto index and the variance of the log-normal distribution are related to each other.

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1 Introduction

Log-normal distributions are frequently observed not only in natural phenomena but also in social phenomena. For representative example, the probability density function $P(x)$ of personal income or firm size x is considered to obey the log-normal distribution [1]–[2]

$$P_{\mathrm{LN}}(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\ln^2\left(x/\bar{x}\right)}{2\sigma^2}\right] \tag{1}$$

in the middle scale region. [1] Here $\bar{x}$ is a mean value and σ^2 is a variance.

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[1] In the large scale region, personal income distributions and firm size those follow power-law [3]. We discuss this in Section 2. In the low scale region, several distributions are proposed (see Refs. [4]–[5] for instance). We do not discuss them in this study.

The simplest model which describes the log-normal distribution is the pure multiplicative stochastic process defined by

$$x(t+1) = R(t) \ x(t) \ , \tag{2}$$

where $R(t)$ is a positive random variable. By applying this process iteratively, we obtain

$$x(t) = R(t-1) \ R(t-2) \ \cdots \ R(0) \ x(0) \ . \tag{3}$$

The logarithm of this equation is

$$\log x(t) = \log R(t-1) + \log R(t-2) + \cdots + \log R(0) + \log x(0) \ . \tag{4}$$

If $\log x(0)$ is negligible compare to $\log x(t)$ in the limit $t \to \infty$ and

$\log R(i)$ $(i = 0, 1, \cdots, t-1)$ are independent probability variables, $\log x(t)$ follows the normal distribution in the limit. As a result, this model derives the stationary log-normal distribution $P_{\mathrm{LN}}(x)$.

Eq. (2) means that the distribution of the growth rate does not depend on $x(t)$. This is known as Gibrat's law [1] that the conditional probability density function of the growth rate $Q(R|x_1)$ is independent of the initial value x_1:

$$Q(R|x_1) = Q(R) \ . \tag{5}$$

Here x_1 and x_2 are two successive incomes, assets, sales, profits, the numbers of employees and so forth. The growth rate R is defined as $R = x_2/x_1$ and $Q(R|x_1)$ as

$$Q(R|x_1) = \frac{P_{1R}(x_1, R)}{P(x_1)} \tag{6}$$

by using the probability density function $P(x_1)$ and the joint probability density function $P_{1R}(x_1, R)$.

As far as firm sizes in the middle scale region, however, it is reported that the growth rate distributions do not follow the Gibrat's law (5) (see Refs. [6]–[8] for instance). The pure multiplicative stochastic process model (2) cannot be applied for explaining the log-normal distribution in the middle scale region. Instead, we have shown that the log-normal distribution can be derived by using no model such as the pure multiplicative stochastic process [9]. In the derivation, two laws are employed which are observed in profits data of Japanese firms. One is the law of detailed balance which is observed in a stable economy [10]. The other is a Non-Gibrat's law which describes a statistical dependence in the growth rate of the past value [9].

In Ref. [9], we kinematically derived the static log-normal distribution under the detailed balance, and analyzed empirical data in a single term 2003–2004 to confirm the derivation. In this study, we show data analyses not only in 2003–2004 but also in 2004–2005 and 2003–2005 to confirm the derivation more firmly. After that, we dynamically derive the log-normal distribution by replacing the detailed balance with the detailed quasi-balance proposed in Ref. [11]. By this procedure, the log-normal distribution is described as quasi-static.

2   Static log-normal distribution under the detailed balance

In this section, we briefly review the study in Ref. [9]. After that, we empirically confirm the analytic result by added data analyses. We employ profits data of Japanese firms in 2003, 2004 and 2005 which are available on the database "CD Eyes 50" published in 2005 and 2006 by TOKYO SHOKO RESEARCH, LTD. [12].

Figure 1 shows the joint probability density function $P_{12}(x_1, x_2)$ of all firms in the database, the profits of which in 2003 (x_1)–2004 (x_2) exceeded 0, $x_1 > 0$ and $x_2 > 0$. The number of the firms is "227,132". Similarly, figures 2–3 show the joint probability density functions of the profits in 2004 (x_1)–2005 (x_2) and 2003 (x_1)–2005 (x_2), and the number of the firms is "232,497" and "197,867", respectively. From Figs. 1–3, we approximately confirm the detailed balance which is time-reversal symmetry $(x_1 \leftrightarrow x_2)$ of $P_{12}(x_1, x_2)$ [10]:

$$P_{12}(x_1, x_2) = P_{12}(x_2, x_1) . \tag{7}$$

Figure 4 shows probability density functions of profits in 2003–2005. The distributions are almost stable and following power-law is observed in the large scale region

$$P(x) = Cx^{-\mu-1} \quad \text{for} \quad x > x_0 , \tag{8}$$

where x_0 is a certain threshold. This power-law is called Pareto's law [3] and the exponent μ is named Pareto index. Notice that Pareto's law does not hold below the threshold x_0. The purpose of this section is to exhibit the distribution in the middle scale region under the detailed balance (7).

In order to identify a statistical dependence in the growth rate of the past value, we examine the probability density faction of the profits growth rate in 2003–2004 firstly. In the database, we divide the range of x_1 into logarithmically equal bins as $x_1 \in 4 \times [10^{1+0.2(n-1)}, 10^{1+0.2n}]$ thousand yen with $n = 1, 2, \cdots, 20$. Figure 5 shows the probability density functions for $r = \log_{10} R$ in the case of $n = 1, \cdots, 5$, $n = 6, \cdots, 10$, $n = 11, \cdots, 15$ and $n = 16, \cdots, 20$. The number of the firms in Fig. 5 is "$22,005$", "$89,507$", "$85,020$" and "$24,203$", respectively.

From Fig. 5, we approximate $\log_{10} q(r|x_1)$ by linear functions of r:

$$\log_{10} q(r|x_1) = c(x_1) - t_+(x_1)\, r \quad \text{for } r > 0 , \tag{9}$$

$$\log_{10} q(r|x_1) = c(x_1) + t_-(x_1)\, r \quad \text{for } r < 0 . \tag{10}$$

Here $q(r|x_1)$ is the probability density function for r, which is related to that for R by $Q(R|x_1) = \log_{10} q(r|x_1) - r - \log_{10}(\ln 10)$. These approximations (9)–(10) are expressed as tent-shaped exponential forms as follows

$$Q(R|x_1) = d(x_1)\, R^{-t_+(x_1)-1} \quad \text{for } R > 1 , \tag{11}$$

$$Q(R|x_1) = d(x_1)\, R^{+t_-(x_1)-1} \quad \text{for } R < 1 , \tag{12}$$

where $d(x_1) = 10^{c(x_1)}/\ln 10$. Furthermore, figure 5 shows that the dependence of $c(x_1)$ on x_1 is negligible for $n = 9, \cdots, 20$. We assess the validity of these approximations against the results.

Figure 6 represents the dependence of $t_\pm(x_1)$ on the lower bound of each bin $x_1 = 4 \times 10^{1+0.2(n-1)}$. For $n = 17, \cdots, 20$, $t_\pm(x_1)$ hardly responds to x_1. This means that Gibrat's law (5) holds only in the large scale region of profits $(x > x_0)$. [2] In contrast, $t_+(x_1)$ linearly increases and $t_-(x_1)$ linearly decreases symmetrically with $\log_{10} x_1$ for $n = 9, \cdots, 13$. From Fig. 6, the slops are described as [13]

$$t_\pm(x_1) = t_\pm(x_0) \pm \alpha \; \ln \frac{x_1}{x_0} \; . \tag{13}$$

The parameters are estimated as follows: $\alpha \sim 0$ for $x_1 > x_0$, $\alpha \sim 0.14$ for $x_{\min} < x_1 < x_0$, $x_0 = 4 \times 10^{1+0.2(17-1)} \sim 63,000$ thousand yen and $x_{\min} = 4 \times 10^{1+0.2(9-1)} \sim 1,600$ thousand yen. Notice that approximations (9)–(10) uniquely fix the expression of $t_\pm(x_1)$ under the detailed balance [9]. This derivation is included in the proof in the next section. We call Eqs.(11)–(13) Non-Gibrat's law.

For $n = 9, \cdots, 20$, the dependence of $d(x_1)$ on x_1 is negligible. In this case, the Non-Gibrat's law determines the probability density function of profits as follows

$$P(x_1) = C x_1^{-(\mu+1)} \; e^{-\alpha \ln^2 \frac{x_1}{x_0}} \qquad \text{for} \;\; x_1 > x_{\min} \; , \tag{14}$$

where $t_+(x_0) - t_-(x_0) \sim \mu$ [14]. This is the power-law in the large scale region $(x_1 > x_0)$ and the log-normal distribution in the middle scale region $(x_{\min} < x_1 < x_0)$. The relations between parameters σ, $\bar{x}$ in Eq. (1) and α, μ, x_0 are given by

$$\alpha = \frac{1}{2\sigma^2} \; , \quad \mu = \frac{1}{\sigma^2} \ln \frac{x_0}{\bar{x}} \; . \tag{15}$$

Figure 7 shows that the distribution (14) fits with the empirical data consistently. Notice that the distribution cannot fit with the empirical data, if α is different from the value estimated in Fig. 6 ($\alpha = 0.10$ or $\alpha = 0.20$ for instance).

These empirical data analyses are not restricted in the single term 2003–2004. Figures 8–10 show similar empirical data analyses in 2004–2005. The number of the firms in Fig. 8 is "20,669", "89,064", "88,651" and "26,887", respectively. Furthermore, empirical data analyses in 2003–2005 are shown in Figs. 11–13. The number of the firms in Fig. 11 is "18,101", "76,050", "75,451" and "22,456", respectively. In Figs. 9, 12, the parameters α, x_0 and $x_{\min}$ in 2004–2005 or 2003–2005 are estimated to be same values in 2003–2004. The distribution (14) in 2004–2005 or 2003–2005 also fits with the empirical data consistently (Figs. 10, 13).

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[2]Fujiwara et al. [10] prove that Pareto's law (8) is derived from the Gibrat's law (5) valid only in the large scale region and the detailed balance (7). In the derivation, linear approximations (9)–(10) need not be assumed.

3 Quasi-static log-normal distribution under the detailed quasi-balance

In the previous section, the log-normal distribution in the middle scale region is exhibited by the Non-Gibrat's law under the detailed balance. The resultant profits distribution is empirically confirmed in data analyses of Japanese firms in 2003–2004, 2004–2005 and 2003–2005. The profits distribution (14) is static, because the derivation is based on the detailed balance (7) which is static time-reversal symmetry (Fig. 1–3).

On the other hand, we have derived Pareto's law with annually varying Pareto index under the detailed quasi-balance [11]:

$$P_{12}(x_1, x_2) = P_{12}\left(\left(\frac{x_2}{a}\right)^{1/\theta}, a\ x_1^{\theta}\right) . \tag{16}$$

It is assumed that, in the quasi-static system, the joint probability density function has "$a\ x_1^{\theta} \leftrightarrow x_2$" symmetry where θ is a slope of a regression line:

$$\log_{10} x_2 = \theta\ \log_{10} x_1 + \log_{10} a . \tag{17}$$

The detailed balance (7) has the special symmetry $\theta = a = 1$. Because the detailed quasi-balance (16) is imposed on the system, θ is related to a as follows:

$$\theta = 1 - \frac{2}{\Gamma} \log_{10} a . \tag{18}$$

Here 10^{Γ} is a sufficient large value compared to the upper bound in which θ and a are estimated.

In Ref. [11], these results have been empirically confirmed by employing data on the assessed value of land [15]–[16] in 1983–2005. In the derivation, we have used the Gibrat's law (5) valid only in the large scale region without linear approximations (9)–(10). The purpose of this section is to show that the approximations uniquely fix a Non-Gibrat's law under the detailed quasi-balance. After that, we identify the quasi-static distribution not only in the large scale region but also in the middle scale region.

By using the relation $P_{12}(x_1, x_2)dx_1 dx_2 = P_{1R}(x_1, R)dx_1 dR$ under the change of variables $(x_1, x_2) \leftrightarrow (x_1, R)$, these two joint probability density functions are related to each other

$$P_{1R}(x_1, R) = a\ x_1^{\theta}\ P_{12}(x_1, x_2) , \tag{19}$$

where we use a modified growth rate $R = x_2/(a\ x_1^{\theta})$. From this relation, the detailed quasi-balance (16) is rewritten as

$$P_{1R}(x_1, R) = R^{-1} P_{1R}\left(\left(\frac{x_2}{a}\right)^{1/\theta}, R^{-1}\right) . \tag{20}$$

Substituting $P_{1R}(x_1, R)$ for $Q(R|x_1)$ defined in Eq. (6), the detailed quasi-balance is reduced to be

$$\frac{P(x_1)}{P((x_2/a)^{1/\theta})} = \frac{1}{R} \frac{Q(R^{-1}|\,(x_2/a)^{1/\theta})}{Q(R|x_1)} . \tag{21}$$

In the quasi-static system, we also assume that $Q(R|x_1)$ follows the tent-shaped exponential forms (11)–(12). Under the approximations, the detailed quasi-balance is expressed as

$$\frac{\tilde{P}(x_1)}{\tilde{P}((x_2/a)^{1/\theta})} = R^{t_+(x_1)-t_-((x_2/a)^{1/\theta})+1} \tag{22}$$

for $R > 1$. Here we use the notation $\tilde{P}(x) = P(x)d(x)$. By expanding Eq.(22) around $R = 1$, the following differential equation is obtained:

$$\theta \left[t_+(x) - t_-(x) + 1\right]\tilde{P}(x) + x\tilde{P}'(x) = 0 , \tag{23}$$

where x denotes x_1. The same equation is obtained for $R < 1$. The solution is expressed as

$$\tilde{P}(x) = Cx^{-\theta} e^{-G(x)} , \tag{24}$$

where $\theta \left[t_+(x) - t_-(x)\right] \equiv g(x)$ and $\int dx \, g(x)/x \equiv G(x)$.

In order to make the solution (24) around $R = 1$ satisfy Eq. (22), the following equation must be valid for all R:

$$-G(x) + G(R^{1/\theta}x) = \left[t_+(x) - t_-(R^{1/\theta}x)\right] \ln R . \tag{25}$$

The derivative of this equation with respect to x is

$$-\frac{g(x)}{x} + \frac{g(R^{1/\theta}x)}{x} = \left[t_+'(x) - R^{1/\theta} \, t_-'(R^{1/\theta}x)\right] \ln R . \tag{26}$$

By expanding Eq. (26) around $R = 1$, the following differential equations are obtained:

$$x \left[t_+''(x) + t_-''(x)\right] + t_+'(x) + t_-'(x) = 0 , \tag{27}$$

$$(1 - 3\theta) \, t_+'(x) + (2 - 3\theta) \, t_-'(x) + 3(1 - \theta) \, x \, t_+''(x) + 3(2 - \theta) \, x \, t_-''(x)$$
$$+ x^2 \left[t_+^{(3)}(x) + 2 \, t_-^{(3)}(x)\right] = 0 . \tag{28}$$

The solutions are given by

$$t_+(x) = -\frac{C_{-2}}{2} \ln^2 x + (C_{+1} - C_{-1}) \ln x + (C_{+0} - C_{-0}) , \tag{29}$$

$$t_-(x) = \frac{C_{-2}}{2} \ln^2 x + C_{-1} \ln x + C_{-0} . \tag{30}$$

In order to make these solutions satisfy Eq. (25), the coefficients C_{-2} and C_{+1} must be 0 . As a result, $t_\pm(x)$ under the detailed quasi-balance is uniquely fixed as

$$t_\pm(x) = t_\pm(x_0) \pm \alpha \, \ln \frac{x}{x_0} . \tag{31}$$

This is the same expression under the detailed balance.

By using the Non-Gibrat's law (31), probability density functions $P_1(x_1)$, $P_2(x_2)$ are uniquely reduced to be

$$P_1(x_1) = C_1 \, x_1^{-\mu_1 - 1} \, \exp\left[-\theta \, \alpha \ln^2 \frac{x_1}{x_0}\right], \tag{32}$$

$$P_2(x_2) = C_2 \, x_2^{-\mu_2 - 1} \, \exp\left[-\theta \, \alpha \ln^2 \frac{(x_2/a)^{1/\theta}}{x_0}\right] \tag{33}$$

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with

$$\frac{\mu_1 + 1}{\mu_2 + 1} \quad = \quad \theta \tag{34}$$

in the large or middle scale region where the dependence of $d(x_1)$ on x_1 is negligible.

Here we consider two log-normal distributions in the middle scale region:

$$P_{\mathrm{LN}_1}(x_1) = \frac{1}{x_1\sqrt{2\pi\sigma_1{}^2}} \exp\left[-\frac{\ln^2(x_1/\bar{x}_1)}{2\sigma_1{}^2}\right] \ , \tag{35}$$

$$P_{\mathrm{LN}_2}(x_2) = \frac{1}{x_2\sqrt{2\pi\sigma_2{}^2}} \exp\left[-\frac{\ln^2(x_2/\bar{x}_2)}{2\sigma_2{}^2}\right] \ . \tag{36}$$

By comparing Eqs. (32)–(33) to (35)–(36), we identify

$$\theta\,\alpha = \frac{1}{2\sigma_1{}^2} \ , \quad \mu_1 = \frac{1}{\sigma_1{}^2}\ln\frac{x_0}{\bar{x}_1} \ , \tag{37}$$

$$\frac{\alpha}{\theta} = \frac{1}{2\sigma_2{}^2} \ , \quad \mu_2 = \frac{1}{\sigma_2{}^2}\ln\frac{ax_0{}^\theta}{\bar{x}_1} \ . \tag{38}$$

Consequently, the relation between σ_1, σ_2 and θ is expressed as

$$\frac{\sigma_2}{\sigma_1} = \theta \ . \tag{39}$$

This is the equation which quasi-static log-normal distributions satisfy.

4   Conclusion

In this study, by employing profits data of Japanese firms, we have exhibited the static log-normal distribution of profits in the middle scale region from a Non-Gibrat's law under the detailed balance. In the derivation, we have adopted two approximations observed in the database. One is that the probability density function of the profits growth rate is described as a tent-shaped exponential function. The other is that the value of the origin of the growth rate distribution divided into bins is constant. These approximations are confirmed in 2003–2004, 2004–2005 and 2003–2005. The resultant profits distribution fits with the empirical data consistently. This guarantees the validity of the approximations.

This static discussion has been applied to a quasi-static system. We have derived a quasi-static log-normal distribution in the middle scale region from a Non-Gibrat's law under the detailed quasi-balance. In the derivation, we have also assumed two approximations confirmed in the database. Even in the quasi-static system, the approximations uniquely fix the Non-Gibrat's law to be the same expression under the detailed balance. The resultant distribution is power-law with varying Pareto index in the large scale region and the quasi-static log-normal distribution in the middle scale region. Notice that not only the change of Pareto index μ but also the change of the variance of the log-normal distribution σ depends on a parameter of the

detailed quasi-balance θ. In other words, μ and σ are related to each other. This phenomenon is suggested in Ref. [17].

First issue in the future is to confirm this quasi-static derivation by data analyses. For this aim, long-term economic data must be investigated in the middle scale region where dynamical transitions are observed. We believe that the analytic results in this study are confirmed in such a database, if two approximations assumed in this paper are applicable.

Acknowledgments

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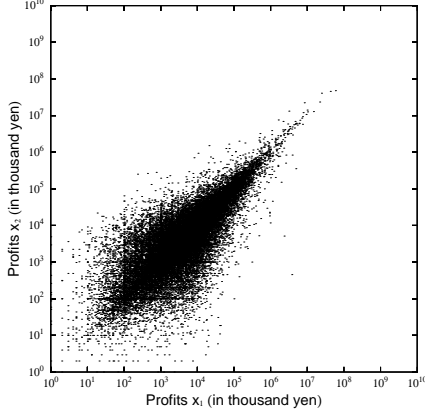
Figure 1: The scatter plot of all firms in the database, the profits of which in 2003 (x_1) and 2004 (x_2) exceeded 0, $x_1 > 0$ and $x_2 > 0$. The number of the firms is "227,132".
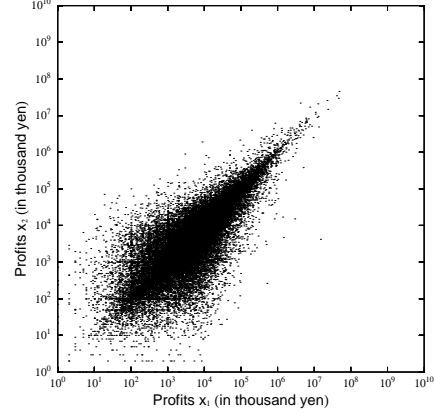
Figure 2: The scatter plot of all firms in the database, the profits of which in 2004 (x_1) and 2005 (x_2) exceeded 0, $x_1 > 0$ and $x_2 > 0$. The number of the firms is "232,497 ".
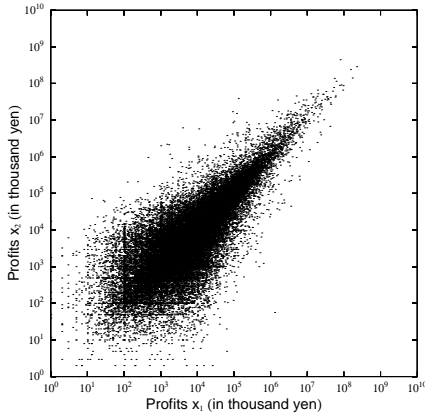
Figure 3: The scatter plot of all firms in the database, the profits of which in 2003 (x_1) and 2005 (x_2) exceeded 0, $x_1 > 0$ and $x_2 > 0$. The number of the firms is "197,867 ".
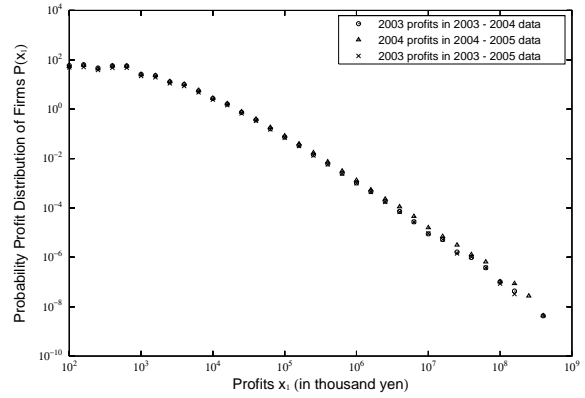
Figure 4: Probability density functions of profits in 2003–2005. In the large scale region, Pareto's law is observed. Pareto index is estimated to be nearly 1.
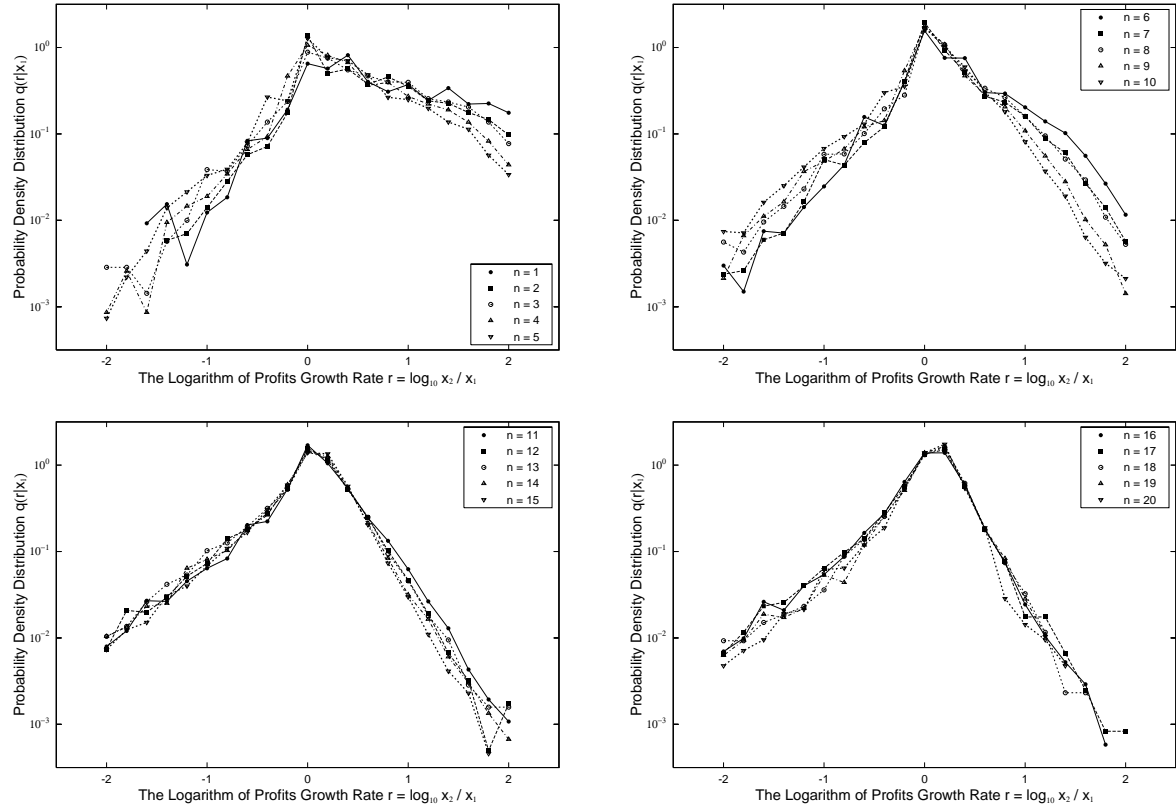
Figure 5: The probability density distribution $q(r|x_1)$ of the log profits growth rate $r = \log_{10} x_2/x_1$ from 2003 to 2004 for $n = 1, 2, \cdots, 20$.
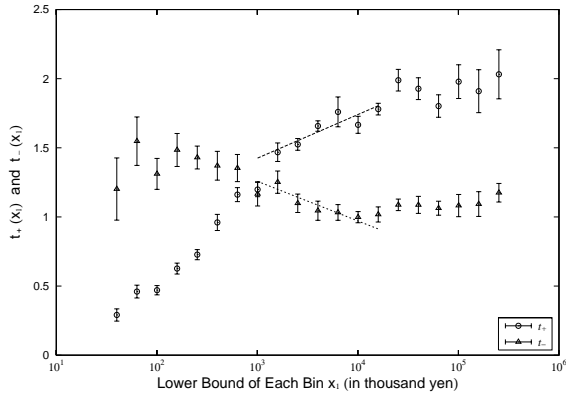
Figure 6: The relation between the lower bound of each bin x_1 and $t_{\pm}(x_1)$ with respect to the profits growth rate from 2003 to 2004. From the left, each data point represents $n = 1, 2, \cdots, 20$.
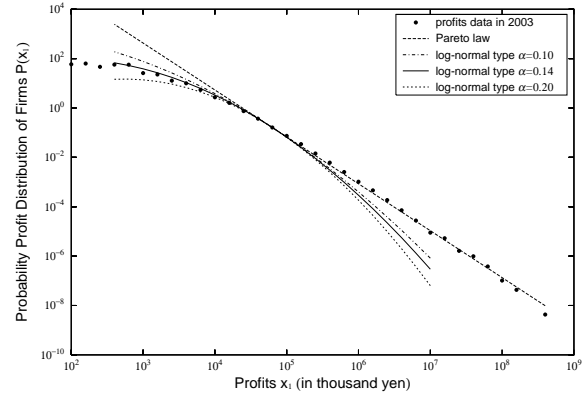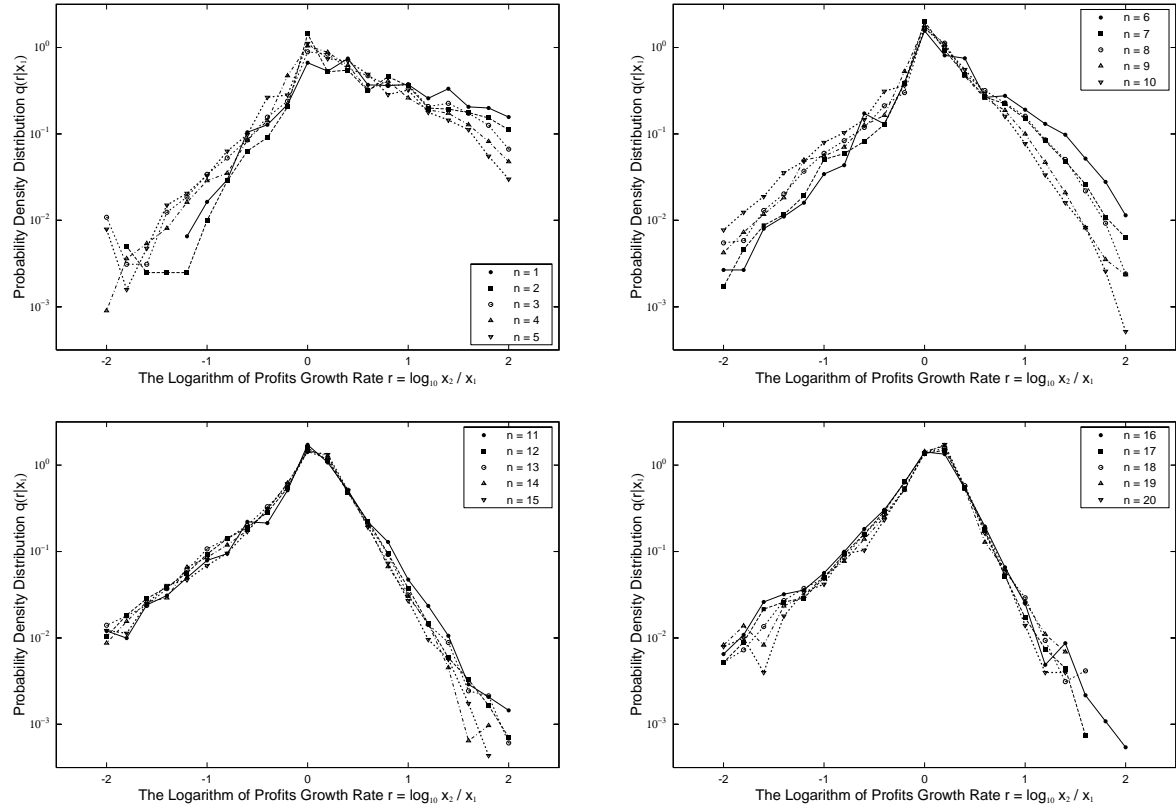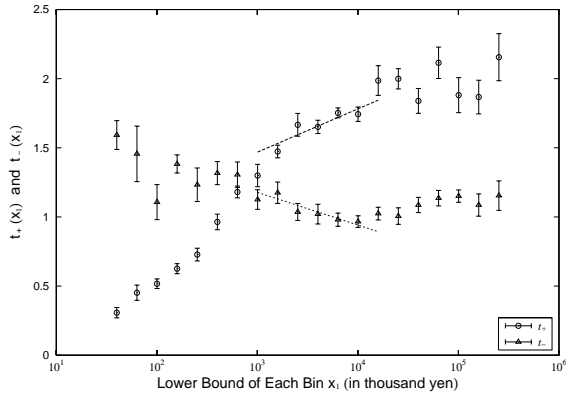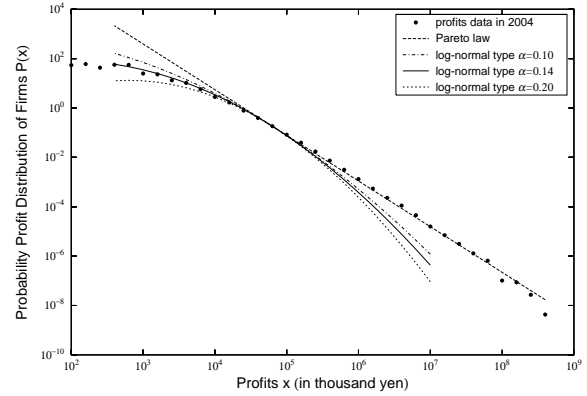
Figure 7: The probability distribution function of profits $P(x_1)$ for firms, the profits of which in 2003 (x_1) and 2004 (x_2) exceeded 0, $x_1 > 0$ and $x_2 > 0$.
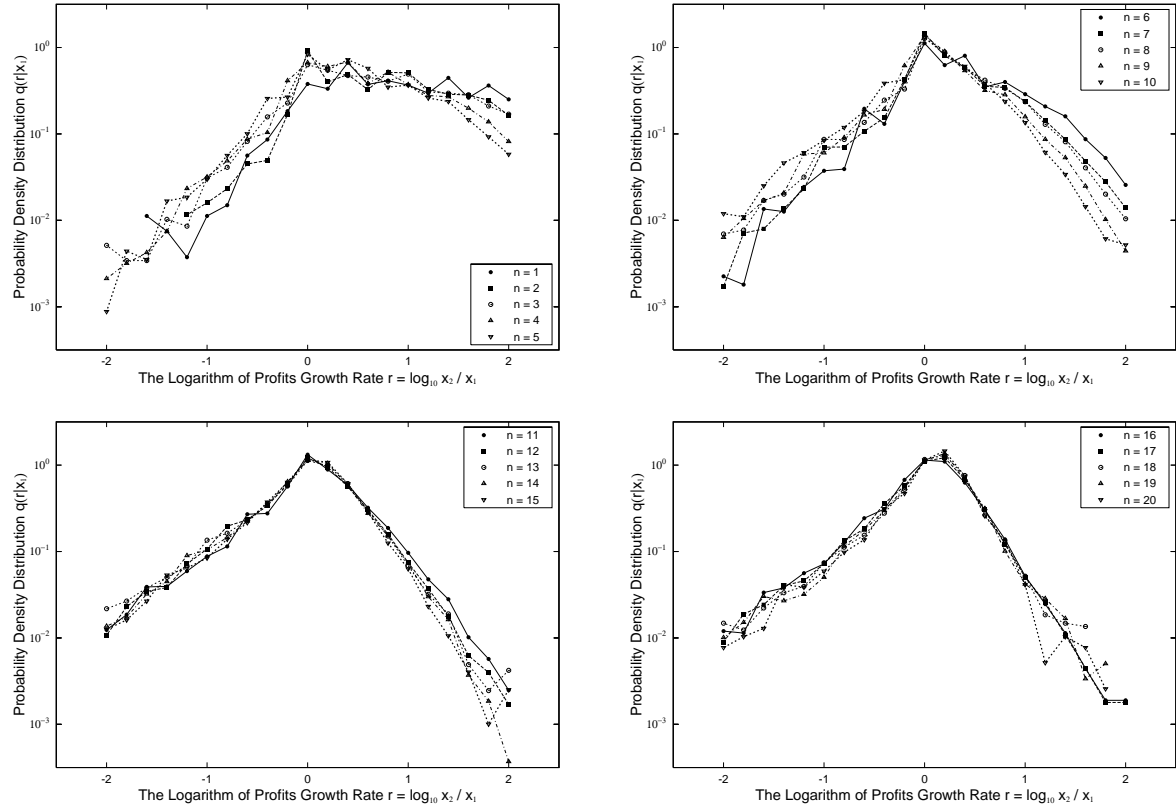
Figure 8: The probability density distribution $q(r|x_1)$ of the log profits growth rate $r = \log_{10} x_2/x_1$ from 2004 to 2005 for $n = 1, 2, \cdots, 20$.

Figure 9: The relation between the lower bound of each bin x_1 and $t_{\pm}(x_1)$ with respect to the profits growth rate from 2004 to 2005. From the left, each data point represents $n = 1, 2, \cdots, 20$.

Figure 10: The probability distribution function of profits $P(x_1)$ for firms, the profits of which in 2004 (x_1) and 2005 (x_2) exceeded 0, $x_1 > 0$ and $x_2 > 0$.

Figure 11: The probability density distribution $q(r|x_1)$ of the log profits growth rate $r = \log_{10} x_2/x_1$ from 2003 to 2005 for $n = 1, 2, \cdots, 20$.
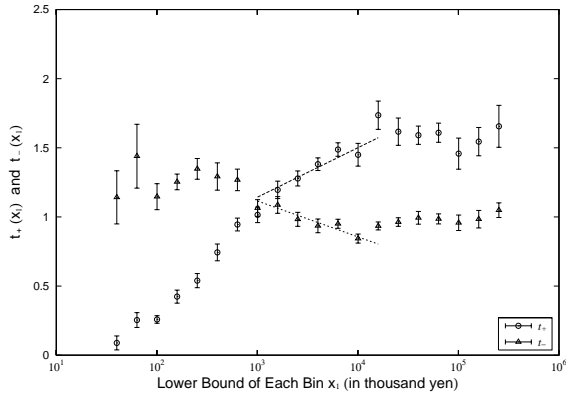

Figure 12: The relation between the lower bound of each bin x_1 and $t_{\pm}(x_1)$ with respect to the profits growth rate from 2003 to 2005. From the left, each data point represents $n = 1, 2, \cdots, 20$.
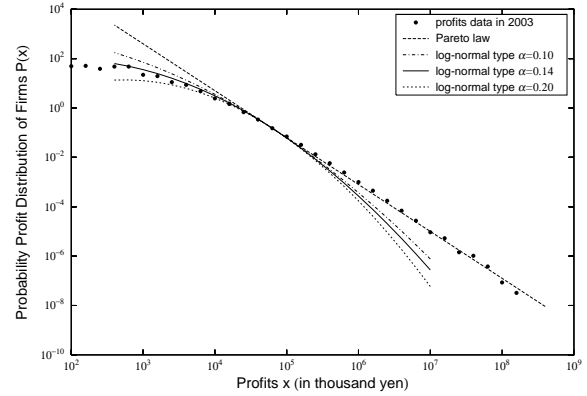
Figure 13: The probability distribution function of profits $P(x_1)$ for firms, the profits of which in 2003 (x_1) and 2005 (x_2) exceeded 0, $x_1 > 0$ and $x_2 > 0$.

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